

Spinodal effect in the natural inflation model

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Recently, Cormier and Holman pointed out that fluctuations of an inflaton field ϕ are significantly enhanced in the model of *spinodal inflation* with a potential $V(\phi)$ for which the second derivative $V^{(2)}(\phi)$ changes sign. As an application of this model, we investigate particle production in the natural inflation model with a potential $V(\phi) = m^4 [1 + \cos(\phi/f)]$ by making use of the Hartree approximation. For typical mass scales $f \sim m_{\text{pl}} \sim 10^{19}\text{GeV}$, and $m \sim m_{\text{GUT}} \sim 10^{16}\text{GeV}$, we find that growth of fluctuations relevantly occurs for the initial value of inflaton $\phi(0) \lesssim 0.1m_{\text{pl}}$. Especially for $\phi(0) \lesssim 10^{-6}m_{\text{pl}}$, maximum fluctuations are so large that secondary inflation takes place by produced fluctuations. In this case, the achieved number of e -folding becomes much larger than in the case where an effect of spinodal instability is neglected.

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I. INTRODUCTION

The concept of inflationary cosmology is very attractive to describe the early stage of the universe [1]. It not only solves horizon, flatness, and monopole problems the standard big bang cosmology has to face, but also provides large-scale density perturbations required for the structure formation [2]. Inflation takes place while a scalar field ϕ called *inflaton* slowly rolls down toward the minimum of its potential $V(\phi)$. The inflationary period ends when the inflaton field begins to oscillate around the minimum of its potential. The elementary particles can be efficiently produced by a nonperturbative process called *preheating* [3,4] in the oscillating stage of inflaton. Then these particles decay to other lighter particles and thermalize the universe.

So far, there are various models of inflation. These different kinds of models can be classified in the following way [5]. The first class is the “large field” model, in which the initial value of inflaton is large and rolls down toward the potential minimum. Chaotic inflation [6] is one of the representative models of this class. The second class is the “small field” model, in which inflaton is small initially and slowly evolves toward the potential minimum at larger values of ϕ . New inflation [7] and natural inflation [8] are good examples of this model. In the first model, the second derivative of the potential $V^{(2)}(\phi)$ usually takes positive values, but in the second model, $V^{(2)}(\phi)$ can change

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sign during inflation. The third one is the hybrid inflation model [9], where the inflaton field has a large amount of potential energy at the minimum of its potential, while the vacuum energy is almost zero at the end of inflation in the first and second inflation models.

Recently, Cormier and Holman [10] have pointed out that fluctuations of inflaton can grow nonperturbatively during an inflationary stage in the second model of inflation when $V^{(2)}(\phi)$ is negative. This idea is remarkable in the sense that particles are effectively produced by negative instability even in the slow rolling stage of inflaton. They call this kind of model *spinodal inflation*, and investigated the nonperturbative evolution of the inflaton field making use of the Hartree approximation in the toy model with a potential $V(\phi) = \frac{3m^4}{2\lambda} - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$. It was found that low momentum modes of fluctuations are mainly enhanced, and the evolution of the system can be described effectively by two classical scalar fields. Especially when ϕ is close to zero initially, the amount of the maximum fluctuation becomes so large that secondary inflation occurs by the effect of produced particles. It was suggested that ordinary prediction of scale invariance of density perturbations generated during inflation would be modified by taking into account this *spinodal effect*.

As for one example of spinodal inflation, we draw attention to the natural inflation model which was originally proposed by Freese et al. [8]. This model is characterized by pseudo Nambu-Goldstone bosons (PNGB) which appear when an approximate global symmetry is spontaneously broken. The PNGB potential is expressed as $V(\phi) = m^4 [1 + \cos(\phi/f)]$, where f and m are two mass scales which characterize the shape of the potential. Considering PNGB as the candidate of inflaton, f and m are constrained by the requirement of sufficient inflation and primordial density perturbations observed by the Cosmic Background Explorer (COBE) satellite. In the case where the effect of spinodal instability is neglected, these mass scales are found to be $f \sim m_{\text{pl}} \sim 10^{19}\text{GeV}$, and $m \sim m_{\text{GUT}} \sim 10^{16}\text{GeV}$ respectively. While other inflation models require extremely weak coupling λ in order to satisfy the constraint of density perturbations (for example, in the chaotic inflation model with self coupling potential, $\lambda \lesssim 10^{-13}$), the PNGB inflation model is preferable in the sense that two mass scales arise naturally in particle physics models. Furthermore, this model has an advantage in the analysis of the spinodal effect. When fluctuations of inflaton grow significantly, higher order terms of fluctuations play a relevant role for the evolution of the system. As compared with other more complicated spinodal models such as new inflation, we can handle these higher order terms in an analytic way in the natural inflation model. What we are concerned with is how the efficient particle production during the natural inflation would modify the dynamics of the system. As we will show later, secondary inflation due to fluctuations

pointed out in Ref. [10] appears and the evolution is drastically changed if the initial value of inflaton is close to zero.

This paper is organized as follows. In the next section, basic equations based on the Hartree approximation are introduced in the natural inflation model. In Sec. III, we study how the fluctuation of inflaton is generated during the inflationary stage. We will show that this effect can significantly alter the dynamics of inflation. We present our discussions and conclusions in the final section.

II. BASIC EQUATIONS

The model we consider is

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right], \quad (2.1)$$

where $\kappa^2/8\pi \equiv G = m_{\text{pl}}^{-2}$ is Newton's gravitational constant, R is a scalar curvature, and ϕ is a minimally coupled inflaton field. In this paper, we adopt a potential which is the so called natural inflation type

$$V(\phi) = m^4 \left[1 + \cos \left(\frac{\phi}{f} \right) \right], \quad (2.2)$$

where two mass scales m and f characterize the height and width of the potential, respectively. The typical mass scales are of the order $f \sim m_{\text{pl}} \sim 10^{19}$ GeV and $m \sim m_{\text{GUT}} \sim 10^{16}$ GeV for the success of the scenario of the natural inflation [8]. The inflaton field is initially located in the region of $0 < \phi(0) < \pi f$, and inflation takes place while inflaton evolves slowly toward the minimum of its potential at $\phi = \pi f$. In order to obtain the sufficient inflation by which the number of e -folding exceeds $N \sim 60$, the initial value of inflaton is required to be close to $\phi = 0$. For example, in the case of $f = 10^{19}$ GeV, we need $\phi(0) \lesssim 0.5 m_{\text{pl}}$. The value of ϕ when inflation ends ($= \phi_F$) depends on the scale f , but ϕ_F is close to the value πf for the typical values of $f \sim m_{\text{pl}}$ [8]. When inflaton begins to oscillate around $\phi = \pi f$, the system enters a reheating stage.

For the potential (2.2), we find that $V^{(2)}(\phi)$ is negative when inflaton evolves in the region of $0 < \phi < \pi f/2$. This leads to the enhancement of fluctuations by spinodal instability in the realistic initial value of ϕ . After the inflaton field passes through $\phi = \pi f/2$ where $V^{(2)}(\phi)$ changes sign, fluctuations of inflaton no longer grow. Hence an important point for the development of fluctuations is the initial value of inflaton. The mass scales f and m also affect the evolution of the system.

Let us obtain basic equations in the natural inflation model. We adopt the flat Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad (2.3)$$

where $a(t)$ is the scale factor, and t is the cosmic time coordinate.

We decompose the quantum scalar field $\phi(t, \mathbf{x})$ into its expectation value $\phi_0(t)$ and the quantum fluctuation $\delta\phi(t, \mathbf{x})$ as

$$\phi(t, \mathbf{x}) = \phi_0(t) + \delta\phi(t, \mathbf{x}), \quad (2.4)$$

with

$$\phi_0(t) = \langle \phi(t, \mathbf{x}) \rangle = \frac{\text{Tr} \phi \rho(t)}{\text{Tr} \rho(t)}, \quad (2.5)$$

where $\rho(t)$ is the density matrix which satisfies the Liouville equation:

$$i \frac{d\rho(t)}{dt} = [\mathcal{H}, \rho(t)]. \quad (2.6)$$

\mathcal{H} is the time dependent Hamiltonian. The initial condition for the density matrix should be chosen to describe a local thermal equilibrium state. Given the initial condition, the time evolution of $\rho(t)$ is known from Eq. (2.6). The expectation value of the energy-momentum tensor is evaluated from the time varying density matrix. Our approach is to solve the semiclassical Einstein equations where the source of the gravitational field is the expectation value of the energy-momentum tensor .

The system we consider is out-of-equilibrium and nonperturbative, and there are some approximations which are suitable to describe such a state. In this paper, we adopt the Hartree mean field approximation of the nonequilibrium quantum field theory. This is basically a Gaussian variational approximation to the time dependent density matrix. Related with this issue, several authors [11–13] considered the large N approximation which can deal with contributions beyond leading order. Although it is of interest to examine the differences between two approximations, the analysis based on the large N approximation is left for the future work [14].

Performing the Hartree factorization

$$\begin{aligned} \delta\phi^{2n} &\rightarrow \frac{(2n)!}{2^n(n-1)!} \langle \delta\phi^2 \rangle^{n-1} \delta\phi^2 - \frac{(2n)!(n-1)}{2^n n!} \langle \delta\phi^2 \rangle^n, \\ \delta\phi^{2n+1} &\rightarrow \frac{(2n+1)!}{2^n n!} \langle \delta\phi^2 \rangle^n \delta\phi, \end{aligned} \quad (2.7)$$

and making use of the tadpole condition

$$\langle \delta\phi(t, \mathbf{x}) \rangle = 0, \quad (2.8)$$

the expectation value of the potential $V(\phi_0 + \delta\phi)$ can be written as

$$\langle V(\phi_0 + \delta\phi) \rangle = \sum_{n=0}^{\infty} \frac{1}{2^n n!} \langle \delta\phi^2 \rangle^n V^{(2n)}(\phi_0), \quad (2.9)$$

where $V^{(n)}(\phi) \equiv \delta^n V(\phi)/\delta\phi^n$. Then the equation of the ϕ_0 field yields

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \sum_{n=0}^{\infty} \frac{1}{2^n n!} \langle \delta\phi^2 \rangle^n V^{(2n+1)}(\phi_0) = 0, \quad (2.10)$$

where a dot denotes a derivative with respect to the cosmic time coordinate, $H \equiv \dot{a}/a$ is the Hubble parameter.

Expanding the $\delta\phi$ field by the Fourier modes as

$$\delta\phi = \frac{1}{(2\pi)^{3/2}} \int \left(a_k \delta\phi_k(t) e^{-i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger \delta\phi_k^*(t) e^{i\mathbf{k}\cdot\mathbf{x}} \right) d^3\mathbf{k}, \quad (2.11)$$

we obtain the following equation for the fluctuation:

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left[\frac{k^2}{a^2} + \sum_{n=0}^{\infty} \frac{1}{2^n n!} \langle \delta\phi^2 \rangle^n V^{(2n+2)}(\phi_0) \right] \delta\phi_k = 0, \quad (2.12)$$

where the expectation value $\langle \delta\phi^2 \rangle$ is represented by

$$\langle \delta\phi^2 \rangle = \frac{1}{2\pi^2} \int k^2 |\delta\phi_k|^2 dk. \quad (2.13)$$

The evolution of the scale factor is written as

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{\kappa^2}{3} \left[\frac{1}{2} \dot{\phi}_0^2 + \frac{1}{2} \langle \delta\dot{\phi}^2 \rangle + \frac{1}{2a^2} \langle (\nabla\delta\phi)^2 \rangle + \sum_{n=0}^{\infty} \frac{1}{2^n n!} \langle \delta\phi^2 \rangle^n V^{(2n)}(\phi_0) \right], \quad (2.14)$$

where $\langle \delta\dot{\phi}^2 \rangle$ and $\langle (\nabla\delta\phi)^2 \rangle$ are expressed by

$$\langle \delta\dot{\phi}^2 \rangle = \frac{1}{2\pi^2} \int k^2 |\delta\dot{\phi}_k|^2 dk, \quad (2.15)$$

$$\langle (\nabla\delta\phi)^2 \rangle = \frac{1}{2\pi^2} \int k^4 |\delta\phi_k|^2 dk. \quad (2.16)$$

The quantities of Eqs. (2.13), (2.15), and (2.16) need to be regulated in order to remove the divergences of integrals.

Several authors considered renormalizations by the method of adiabatic regularization [12,13,15,16] and dimensional regularization [17]. The former scheme is based on introducing a large momentum cutoff and subtracting the leading adiabatic orders of the fluctuation terms. The latter is the covariant regularization in which the counter terms do not

depend on the initial state. However, this dimensional regularization has a shortcoming that the energy-momentum tensor has an initial singularity. In this paper, we make use of the scheme of adiabatic regularization as in Ref. [12,13], which is suitable for numerical computations.

In the natural inflation potential (2.2), Eqs. (2.10), (2.12), and (2.14) can be rewritten as

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 - \frac{m^4}{f} F(\langle \delta\phi^2 \rangle) \sin\left(\frac{\phi_0}{f}\right) = 0, \quad (2.17)$$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left[\frac{k^2}{a^2} - \frac{m^4}{f^2} F(\langle \delta\phi^2 \rangle) \cos\left(\frac{\phi_0}{f}\right)\right] \delta\phi_k = 0, \quad (2.18)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3} \left\{ \frac{1}{2} \dot{\phi}_0^2 + \frac{1}{2} \langle \delta\dot{\phi}^2 \rangle + \frac{1}{2a^2} \langle (\nabla\delta\phi)^2 \rangle + m^4 \left[1 + F(\langle \delta\phi^2 \rangle) \cos\left(\frac{\phi_0}{f}\right) \right] \right\}, \quad (2.19)$$

where

$$F(\langle \delta\phi^2 \rangle) \equiv \exp\left(-\frac{\langle \delta\phi^2 \rangle}{2f^2}\right). \quad (2.20)$$

Note that $F(\langle \delta\phi^2 \rangle) = 1$ in the case of $\langle \delta\phi^2 \rangle = 0$. When ϕ_0 is located in the region $0 < \phi_0 < \pi f/2$ initially, the term in the square bracket in Eq. (2.18) is negative for small k at the first stage of inflation. This leads to the enhancement of fluctuations of low momentum modes. Since ϕ_0 increases for $0 < \phi_0 < \pi f$ as is found in Eq. (2.17), the term $\cos(\phi_0/f)$ in Eq. (2.18) gradually decreases and the growth of fluctuations terminates after ϕ_0 becomes larger than $\pi f/2$.

As is found by Eq. (2.19), the evolution of inflaton can be described effectively by two homogeneous fields with the potential

$$V(\phi_0, \sigma) \equiv m^4 \left[1 + \exp\left(-\frac{\sigma^2}{2f^2}\right) \cos\left(\frac{\phi_0}{f}\right) \right], \quad (2.21)$$

where $\sigma \equiv \sqrt{\langle \delta\phi^2 \rangle}$. This potential is depicted in Fig. 1. When fluctuations do not grow relevantly as $\sigma \ll f$, the ϕ_0 field slowly rolls down toward the potential minimum $\phi_0 = \pi f$ almost along the ϕ_0 direction in the usual manner. The inflationary period ends when the ϕ_0 field begins to oscillate around the minimum of its potential. At this stage, the potential energy becomes small and the expansion rate slows down. However, in the case where σ grows to the order of f , the evolution of the system is drastically modified. The inflaton field moves toward the σ direction rather than the ϕ_0 direction in Fig. 1. It reaches the flat region $\sigma \gtrsim 2m_{\text{pl}}$, and secondary inflation occurs there. We investigate for what initial values of ϕ_0 this behavior appears in the next section.

Before analyzing the evolution of the system, we mention the initial conditions for the fluctuation. We should choose a conformal adiabatic vacuum state where the density matrix represents a local thermal equilibrium, which means that the density matrix commutes with the initial conformal Hamiltonian. This corresponds to choose the mode functions $\delta\phi_k$ as

$$\delta\phi_k(0) = \frac{1}{\sqrt{2\omega_k(0)}}, \quad \delta\dot{\phi}_k(0) = [-i\omega_k(0) - H(0)] \delta\phi_k(0), \quad (2.22)$$

with

$$\omega_k^2(0) = k^2 + \mathcal{M}^2(0), \quad \mathcal{M}^2(0) = -\frac{m^4}{f^2} F(\langle\delta\phi^2\rangle) \cos\left(\frac{\phi_0(0)}{f}\right) - \frac{R(0)}{6}, \quad (2.23)$$

where $R(0)$ is the initial scalar curvature, and we set $a(0) = 1$.

In the present model, since ω_k^2 becomes negative for small k , we need to modify the initial frequencies of low momentum modes. Following the approach performed in Ref. [12], we adopt the initial frequency as

$$\omega_k^2(0) = k^2 + \mathcal{M}^2(0) \tanh\left(\frac{k^2 + \mathcal{M}^2(0)}{|\mathcal{M}^2(0)|}\right), \quad (2.24)$$

This initial frequency coincides with the conformal vacuum frequency (2.23) for large k , and becomes positive for small k . This choice of the initial frequency smoothly interpolates between large and small momentum modes. An alternative way is to choose the initial condition as [13]

$$\omega_k^2(0) = \begin{cases} k^2 + |\mathcal{M}^2(0)| & \text{with } k^2 < |\mathcal{M}^2(0)|, \\ k^2 + \mathcal{M}^2(0) & \text{with } k^2 \geq |\mathcal{M}^2(0)|. \end{cases} \quad (2.25)$$

Although there are some subtleties about the choice of initial frequencies, we can numerically check that these different choices have little effect on the evolution of the system. The qualitative properties of the system are the same in either case of Eq. (2.24) or Eq. (2.25).

We investigate the nonperturbative evolution of $\langle\delta\phi^2\rangle$ with the initial condition of Eqs. (2.22) and (2.24) as the semiclassical problem.

III. PARTICLE PRODUCTION IN THE NATURAL INFLATION MODEL

In this section, we study out-of-equilibrium dynamics due to the enhancement of fluctuations in the natural inflation model. Let us first consider the typical case of $f = 10^{19}$ GeV $\sim m_{\text{pl}}$ and $m = 10^{16}$ GeV $\sim 10^{-3}m_{\text{pl}}$. In order to solve the standard cosmological puzzles of the standard big bang cosmology, the number of e -folding

$$N \equiv \ln \left(\frac{a(t_f)}{a(0)} \right), \quad (3.1)$$

where t_f denotes the time when the slow roll period ends, is required to be $N \gtrsim 60$. We need initial values of inflaton as $\phi(0) \lesssim 0.5m_{\text{pl}}$ to obtain $N \gtrsim 60$ in the case where the spinodal effect is not included [8]. For examples, when $\phi(0) = 0.5m_{\text{pl}}$, $N = 71$; and when $\phi(0) = m_{\text{pl}}$, $N = 39$. One may consider that the enhancement of fluctuations would lead to the larger amount of e -folding and relax the constraint of $\phi(0)$ to yield $N \gtrsim 60$. However, this is not the case. Fluctuations do not grow relevantly for $\phi(0) \gtrsim 0.1m_{\text{pl}}$. We depict the evolution of the ϕ_0 field and the fluctuation σ for the case of $\phi(0) = 0.1m_{\text{pl}}$ in Fig. 2. We find that the maximum fluctuation at $\phi_0 = \pi f/2$ is $\sigma_{\text{max}} \approx 10^{-5}m_{\text{pl}}$. Since $\sigma_{\text{max}}^2/(2f^2) \ll 1$ and $F(\langle \delta\phi^2 \rangle)$ is close to unity, the evolution of the ϕ_0 field and the scale factor are almost the same as in the case where the growth of the fluctuation is neglected. The ϕ_0 field evolves toward the potential minimum at $\phi_0 = \pi f$ without being affected by the back reaction effect of produced particles. Inflationary period ends when $mt \approx 4.5 \times 10^4$, at which the value of ϕ_0 is $\phi_0 \approx 3.0f$ [8].

In the case of $\phi(0) \lesssim 0.1m_{\text{pl}}$, the number of e -folding becomes larger with the decrease of $\phi(0)$ since the slow roll period is longer. In addition to this, fluctuations are enhanced more efficiently. In TABLE I, we show numerical values of N and σ_{max} in various cases of $\phi(0)$. The number of e -folding N' where the spinodal effect is neglected is also presented. We find that both N and σ_{max} increase with the decrease of $\phi(0)$ for $10^{-6}m_{\text{pl}} \lesssim \phi(0) \lesssim 10^{-1}m_{\text{pl}}$. The growth of fluctuations continues until the ϕ_0 field reaches $\phi_0 = \pi f/2$. With the decrease of $\phi(0)$, since the period during which the inflaton field moves in the region of $V^{(2)}(\phi) < 0$ becomes longer, this results in the larger amount of the maximum fluctuation. In the case of $10^{-6}m_{\text{pl}} \lesssim \phi(0) \lesssim 10^{-1}m_{\text{pl}}$, numerical calculations show that σ_{max} can be approximately written as a function of $\phi(0)$:

$$\sigma_{\text{max}} \approx \left(\frac{\phi(0)}{10^{-6}m_{\text{pl}}} \right)^{-1} m_{\text{pl}}. \quad (3.2)$$

When $\phi(0) \gtrsim 10^{-5}m_{\text{pl}}$, σ_{max} does not exceed $0.1m_{\text{pl}}$. In this case, since $\langle \delta\phi^2 \rangle/2f^2 \ll 1$ in Eq. (2.20), the back reaction effect due to fluctuations can be neglected. Although the inflaton field moves a bit toward the σ direction in Fig. 1, it slowly rolls down toward the minimum of its potential in the usual manner. As a result, the number of e -folding does not change even taking into account the spinodal effect (see TABLE I).

On the other hand, when $\phi(0) \lesssim 10^{-6}m_{\text{pl}}$, the fluctuation reaches $\sigma_{\text{max}} \gtrsim m_{\text{pl}}$. In this case, produced fluctuations play a relevant role for the evolution of the system. For example, let us consider the case of $\phi(0) = 5.0 \times 10^{-7}m_{\text{pl}}$. As is shown in Fig. 3, the fluctuation reaches the maximum value $\sigma_{\text{max}} \approx 2.3m_{\text{pl}}$ at $mt = 2.5 \times 10^5$, where $\phi_0 = \pi f/2$.

After that, particle production terminates completely because $V^{(2)}(\phi)$ changes sign, and σ decreases by the expansion of the universe. We can see this behavior of inflaton in Fig. 1. In this case, the inflaton field evolves toward the σ direction rather than the ϕ_0 direction, and reaches the region around $\sigma \approx 2.3m_{\text{pl}}$ and $\phi_0 \approx 1.5m_{\text{pl}}$. Since this region of the effective potential (2.21) is flatter than the region of $\sigma \approx 0$ and $\phi_0 \approx 1.5m_{\text{pl}}$, the inflaton field moves slowly for some time. After that, it evolves along the valley around $0 < \sigma/m_{\text{pl}} < 2.3$ and $\phi_0 \approx \pi f$, and finally arrives at the minimum of its potential. The amount of inflation is larger than in the case where the spinodal effect is ignored as is found in TABLE I, because produced fluctuations provide the additional energy density in Eq. (2.19).

This tendency becomes stronger with the decrease of $\phi(0)$. In Fig. 4, we depict the evolution of ϕ_0 and σ fields in the case of $\phi_0(0) = 3.0 \times 10^{-7}m_{\text{pl}}$. In this case, the fluctuation reaches the maximum value $\sigma_{\text{max}} \approx 3.8m_{\text{pl}}$. One of the different points from the $\phi(0) = 5.0 \times 10^{-7}m_{\text{pl}}$ case is that the inflaton field stays in flat regions for a long time: $1 \times 10^6 \lesssim mt \lesssim 7 \times 10^6$. As is seen in Fig. 1, the effective potential $V(\phi_0, \sigma)$ in the region around $\sigma \approx 3.8m_{\text{pl}}$ and $\phi_0 \approx 1.5m_{\text{pl}}$ is very flat. Since $F(\langle \delta\phi^2 \rangle_{\text{max}})$ is much smaller than unity, $V(\phi_0, \sigma)$ takes the almost constant value m^4 . The third terms in Eqs. (2.17) and (2.18) become very small in this region (since the main contributions to the fluctuation are due to low momentum modes, $k^2/a^2 \rightarrow 0$), and the inflaton field evolves very slowly in the flat region $V(\phi_0, \sigma) \approx m^4$. This results in the secondary inflation supported by fluctuations. This behavior was originally pointed out by Cormier and Holman in the model with a potential $V(\phi) = \frac{3m^4}{2\lambda} - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$ [10]. We can expect that their results based on the Hartree approximation generally hold in more complex spinodal type potentials if we choose initial values of inflaton close to zero. When $\phi(0) = 3.0 \times 10^{-7}m_{\text{pl}}$, the secondary inflation continues much longer than the first inflation driven by the potential energy around $\phi = 0$ at $0 \lesssim mt \lesssim 1.3 \times 10^5$ (See Fig. 5). This means that the number of e -folding is modified to be much larger than in the case where particle production due to spinodal instability is neglected. In fact, as is found in TABLE I, the number of e -folding is very large as $N = 22178$. After the secondary inflation ends, the inflaton field rolls down toward the potential minimum around $\phi_0 = \pi f$ and $\sigma = 0$, after which the universe enters the reheating stage.

With the decrease of initial values of inflaton as $\phi(0) \lesssim 10^{-7}m_{\text{pl}}$, since the fluctuation grows as $\sigma \gtrsim 4m_{\text{pl}}$, inflaton reaches the further flat region of the potential $V(\phi_0, \sigma)$. The duration of the secondary inflation becomes very long, and the amount of inflation is enormously large. As long as $0 < \phi_0 < \pi f/2$, the inflaton field is to roll down toward the σ direction, and the secondary inflation continues. However, once ϕ_0 exceeds the value of $\phi_0 = \pi f/2$, σ begins to decrease toward $\sigma = 0$. The secondary inflation never ends in the extreme case of $\phi_0 = 0$ as was pointed out in

Ref. [10].

Next, we investigate the case where m and f are changed. The mass m is constrained by density perturbations observed by COBE. The analytic estimation neglecting particle production due to spinodal instability [8] shows that m is constrained by a function of f as

$$m = \frac{1.7 \times 10^{16}}{b^{1/2}} \left[\frac{m_{\text{pl}}}{f} \sin \left(\frac{\phi(t_f)}{2f} \right) \right]^{1/2} \exp \left(-\frac{15m_{\text{pl}}^2}{8\pi f^2} \right) \text{ GeV}, \quad (3.3)$$

where b is an overall bias factor which ranges $0.7 < b < 1.3$. The term $\sin(\phi(t_f)/2f)$ is typically of the order of unity. In the case where f is of order m_{pl} , m ranges in the region of $10^{15} \text{ GeV} \lesssim m \lesssim 10^{16} \text{ GeV}$. When f is smaller than m_{pl} by one order of magnitude, m decreases significantly because the exponential term in Eq. (3.3) plays a dominant role to determine the mass scale. If we include the effect of spinodal instability, since this would produce the spatial inhomogeneity, the constraint m will be changed. In order to study this issue appropriately, we have to investigate the evolution of metric perturbations during inflation. In the present model, however, since metric perturbations may be enhanced up to the nonlinear level by spinodal instability, the first order perturbation will not give the correct description of physics. Although we do not consider these complex issues in this paper, it would be necessary to include metric perturbations for a complete study of nonperturbative dynamics.

Let us study the growth of fluctuations by changing the scale m for the fixed value of $f = 10^{19} \text{ GeV} \sim m_{\text{pl}}$. Consider the case of $m = 10^{15} \text{ GeV} \sim 10^{-4} m_{\text{pl}}$. When particle creation by spinodal instability is neglected, since the achieved number of e -folding is expressed by

$$N = \frac{16\pi f^2}{m_{\text{pl}}^2} \ln \left[\frac{\sin(\phi(t_f)/2f)}{\sin(\phi(0)/2f)} \right], \quad (3.4)$$

it does not depend on the scale of m . Although it takes more time to terminate inflation for smaller values of m , we obtain $N \gtrsim 60$ for $\phi(0) \lesssim 0.5m_{\text{pl}}$, which is the same as in the $m = 10^{16} \text{ GeV}$ case. As for fluctuations of inflaton, since $\langle \delta\phi^2 \rangle$ is normalized by the square of mass m , the achieved maximum value of σ becomes smaller as m decreases for the same initial value of ϕ . Particle production relevantly occurs for the case of $\phi(0) \lesssim 10^{-3} m_{\text{pl}}$. Numerical calculations show that the maximum fluctuation σ_{max} in the case of $10^{-8} m_{\text{pl}} \lesssim \phi(0) \lesssim 10^{-3} m_{\text{pl}}$ is smaller by two orders of magnitude than in the case of $m = 10^{16} \text{ GeV}$ (see TABLE II). Namely, we find the relation

$$\sigma_{\text{max}} \approx \left(\frac{\phi(0)}{10^{-6} m_{\text{pl}}} \right)^{-1} 10^{-2} m_{\text{pl}}. \quad (3.5)$$

When $\phi(0) \lesssim 10^{-8} m_{\text{pl}}$, σ_{max} exceeds the order of $f \sim m_{\text{pl}}$, and the secondary inflation occurs as in the case of $m = 10^{16} \text{ GeV}$. The number of e -folding becomes larger than in the case where the spinodal effect is neglected, and it

depends on the scale of m . We found that the smaller values of $\phi(0)$ are required for the development of fluctuations with the decrease of m .

Finally we comment on the case where the mass f is changed. The number of e -folding is smaller with the decrease of f , because the potential (2.2) becomes steeper. Consider the case of $f = 5.0 \times 10^{18}$ GeV $\sim 0.5m_{\text{pl}}$ and $m = 10^{15}$ GeV $\sim 10^{-4}m_{\text{pl}}$. Even in the initial value of $\phi(0) = 10^{-1}m_{\text{pl}}$, the number of e -folding is only $N = 33$. In order to lead to the sufficient inflation as $N \gtrsim 60$, we require initial values as $\phi(0) \lesssim 10^{-2}m_{\text{pl}}$. Since the inflaton field rolls down rapidly in the regions of $V^{(2)}(\phi) < 0$ compared with the case of $f = 10^{19}$ GeV, the growth of fluctuations is slower. For example, the maximum fluctuations are $\sigma_{\text{max}} = 6.7 \times 10^{-4}m_{\text{pl}}$ for $\phi(0) = 10^{-5}m_{\text{pl}}$; $\sigma_{\text{max}} = 6.7 \times 10^{-2}m_{\text{pl}}$ for $\phi(0) = 10^{-7}m_{\text{pl}}$ (see TABLE III). These values are smaller than in the case of $f = 10^{19}$ GeV and $m = 10^{15}$ GeV for the same initial values of ϕ . The fluctuation grows up to the nonlinear level for $\phi(0) \lesssim 5 \times 10^{-9}m_{\text{pl}}$, and the secondary inflation also occurs in this case. For the values of f which are not much smaller than the Planck order, we can say that fluctuations are enhanced beyond the perturbative level and can support the total amount of inflation.

IV. CONCLUDING REMARKS AND DISCUSSIONS

In this paper we have investigated the evolution of an inflaton field ϕ in the presence of nonperturbative behavior of fluctuations due to spinodal instability in the natural inflation model. Since the second derivative $V^{(2)}(\phi)$ of the potential $V(\phi) = m^4 [1 + \cos(\phi/f)]$ is negative for the values of $0 < \phi < \pi f/2$, fluctuations of inflaton can grow even during the inflationary phase.

The strength of the excitation of fluctuations σ strongly depends on the initial value of inflaton $\phi(0)$. For typical mass scales $f = m_{\text{pl}}$ and $m = 10^{-3}m_{\text{pl}}$, we have examined the dynamics of the system in various values of $\phi(0)$ by making use of the Hartree approximation. For the values of $\phi(0) \lesssim 0.5m_{\text{pl}}$, we have sufficient inflation as $N \gtrsim 60$ which is required to solve cosmological puzzles of the big bang cosmology. When $\phi(0) \lesssim 0.1m_{\text{pl}}$, fluctuations are relevantly enhanced with the decrease of $\phi(0)$ because duration of spinodal instability becomes longer. Since long wavelength modes of fluctuations are mainly enhanced, the system can be described effectively by two homogeneous fields with potential (2.21). The natural inflation model has the advantage that higher order terms of fluctuations can be handled in an analytic way. With the increase of σ , the term $F(\langle \delta\phi^2 \rangle)$ in Eq. (2.20) decreases from unity. This changes the evolution of the system as is found in Eqs. (2.17)-(2.19). Numerical calculations show that the maximum value of fluctuations is $\sigma_{\text{max}} \approx (\phi(0)/10^{-6}m_{\text{pl}})^{-1} m_{\text{pl}}$ for the case of $10^{-5}m_{\text{pl}} \lesssim \phi(0) \lesssim 10^{-1}m_{\text{pl}}$, which means that

σ_{\max} is less than $\sim 0.1m_{\text{pl}}$. In this case, since σ_{\max} is smaller than the scale f by one order of magnitude, the back reaction effect due to particle production can be neglected. When $\phi(0) \lesssim 10^{-6}m_{\text{pl}}$, however, σ exceeds the scale of f and the dynamics of inflation are altered. Since the effective potential (2.21) is flat in the region of $\sigma \gtrsim 2m_{\text{pl}}$, secondary inflation takes place by produced fluctuations. As compared with the case where the spinodal effect is neglected, the number of e -foldings become much larger. The secondary inflation continues for a long time in the case of $\phi(0) \lesssim 10^{-7}m_{\text{pl}}$. Once inflaton exceeds the value of $\phi_0 = \pi f/2$, it gradually approaches the potential minimum around $\phi_0 = \pi f$ and $\sigma \approx 0$, after which the inflationary period terminates.

If we change two mass scales of m and f , we obtain the smaller maximum fluctuation with the decrease of m and f for the fixed initial values of $\phi(0)$. However, if we choose smaller values of $\phi(0)$ which are close to zero, we find that fluctuations can grow beyond the perturbative level to lead to the secondary inflation. The number of e -folding depends on the scale of m for the fixed values of f and $\phi(0)$ if the spinodal effect is taken into account.

We should comment on some points. The influence due to resonant particle production on cosmic background anisotropies was studied by several authors in the context of preheating [18] and fermion production during inflation [19] in the chaotic inflation model. In the new inflation scenario, it was found that metric perturbations of super-horizon modes are enhanced by spinodal instability when inflaton is initially located around $\phi = 0$ [13]. It is interesting to study how scale invariance of the Harrison-Zel'dovich spectrum would be modified in the present model by making use of the gauge invariant formalisms of metric perturbations [20,21].

As related to the treatment of the back reaction by produced particles, we relied on the Hartree approximation, which is essentially the mean field approximation. In the present model, this has the advantage of being able to deal with higher order contributions of fluctuations analytically beyond perturbation theory. On the other hand, there are other approaches related to back reaction issues. One of them is 2PI formalism by Cornwall et al [22]. Another approach is to add the stochastic noise term due to quantum fluctuations to the field equation, which is based on the closed time path formalism [23]. These formalisms may alter quantitative details obtained in this paper especially when fluctuations are enhanced significantly.

Although we studied the natural inflation model as one example of spinodal inflation, the nonperturbative evolution of fluctuations which leads to secondary inflation would be expected to occur in other spinodal models. As well as in the “small field” models such as the natural inflation and new inflation, in which inflaton is initially small, potentials with spinodal instability appear in the “large field” models such as the higher curvature gravity [24] and the nonminimally

coupled scalar field [25] by performing conformal transformations to the Einstein frame. It is of interest how the dynamics of inflation are modified in these models by taking into account the effect of spinodal instability. These issues are in consideration.

ACKNOWLEDGMENTS

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Note added.

Very recently, Cormier and Holman [26] considered the dynamics of the spinodal instability in the same model as ours. Their results are consistent with our results obtained in this paper.

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TABLE I. The number of e -folding N and the maximum value of the fluctuation σ_{\max} for various initial values of ϕ_0 in the case of $f = 10^{19}$ GeV $\sim m_{\text{pl}}$ and $m = 10^{16}$ GeV $\sim 10^{-3}m_{\text{pl}}$. We also attach the number of e -folding N' where the spinodal effect is neglected. N and σ_{\max} both increase with the decrease of $\phi(0)$ in the case of $10^{-6} < \phi(0)/m_{\text{pl}} < 10^{-1}$. When $\phi(0)/m_{\text{pl}} < 10^{-6}$, the fluctuation grows significantly as $\sigma_{\max}/m_{\text{pl}} > 1$. This leads to secondary inflation, and the amount of inflation drastically increases in the case of $\phi(0)/m_{\text{pl}} < 3 \times 10^{-7}$.

$\phi(0)/m_{\text{pl}}$	N	N'	$\sigma_{\max}/m_{\text{pl}}$
1	39	39	—
0.5	71	71	—
1×10^{-1}	156	156	1.1×10^{-5}
1×10^{-2}	272	272	1.1×10^{-4}
1×10^{-3}	387	387	1.1×10^{-3}
1×10^{-4}	502	502	1.1×10^{-2}
1×10^{-5}	618	618	1.1×10^{-1}
1×10^{-6}	776	734	1.1
5×10^{-7}	1151	771	2.3
3×10^{-7}	22178	797	3.8

TABLE II. The number of e -folding N and the maximum value of the fluctuation σ_{\max} for various initial values of ϕ_0 in the case of $f = 10^{19}$ GeV $\sim m_{\text{pl}}$ and $m = 10^{15}$ GeV $\sim 10^{-4}m_{\text{pl}}$. We also attach the number of e -folding N' where the spinodal effect is neglected. Compared with the case of $f = m_{\text{pl}}$ and $m = 10^{-3}m_{\text{pl}}$, σ_{\max} is smaller by two orders of magnitude for the same initial values of ϕ while N is the same value for $\phi(0)/m_{\text{pl}} > 10^{-5}$. In this case, secondary inflation takes place for $\phi(0)/m_{\text{pl}} < 5 \times 10^{-9}$, which is smaller than in the case of $f = 10^{19}$ GeV and $m = 10^{16}$ GeV.

$\phi(0)/m_{\text{pl}}$	N	N'	$\sigma_{\max}/m_{\text{pl}}$
1×10^{-1}	156	156	—
1×10^{-2}	272	272	—
1×10^{-3}	387	387	1.4×10^{-5}
1×10^{-4}	502	502	1.4×10^{-4}
1×10^{-5}	618	618	1.4×10^{-3}
1×10^{-6}	734	734	1.4×10^{-2}
1×10^{-7}	851	851	1.4×10^{-1}
1×10^{-8}	1043	967	1.4
7×10^{-9}	1234	986	2.0
5×10^{-9}	2512	1004	2.9

TABLE III. The number of e -folding N and the maximum value of the fluctuation σ_{\max} for various initial values of ϕ_0 in the case of $f = 5.0 \times 10^{18}$ GeV $\sim 0.5 m_{\text{pl}}$ and $m = 10^{15}$ GeV $\sim 10^{-4} m_{\text{pl}}$. We also attach the number of e -folding N' where the spinodal effect is neglected. Both N and σ_{\max} take smaller values compared with the case of $f = 10^{19}$ GeV and $m = 10^{15}$ GeV for the same initial value of ϕ . Secondary inflation occurs for initial values of $\phi(0)/m_{\text{pl}} < 5 \times 10^{-9}$.

$\phi(0)/m_{\text{pl}}$	N	N'	$\sigma_{\max}/m_{\text{pl}}$
1×10^{-1}	33	33	—
1×10^{-2}	62	62	—
1×10^{-3}	92	92	6.7×10^{-6}
1×10^{-4}	122	122	6.7×10^{-5}
1×10^{-5}	151	151	6.7×10^{-4}
1×10^{-6}	180	180	6.7×10^{-3}
1×10^{-7}	210	210	6.7×10^{-2}
1×10^{-8}	256	240	6.7×10^{-1}
7×10^{-9}	290	244	9.4×10^{-1}
5×10^{-9}	458	249	1.3

Figure Captions

- FIG. 1: The effective two-field potential $V(\phi_0, \sigma) \equiv m^4 \left[1 + \exp\left(-\frac{\sigma^2}{2f^2}\right) \cos\frac{\phi_0}{f} \right]$ in the natural inflation model. ϕ_0 and σ are normalized by the scale f . In the case where growth of the fluctuation of inflaton is neglected, this is reduced to the one-field potential $V(\phi_0) = m^4 \left[1 + \cos\left(\frac{\phi_0}{f}\right) \right]$. However, when the fluctuation grows significantly and evolves toward the σ direction, the system is described by two fields ϕ_0 and σ .
- FIG. 2: The evolution of ϕ_0 and σ fields for the initial value of $\phi(0) = 0.1m_{\text{pl}}$ in the case of $f = 10^{19} \text{ GeV} \sim m_{\text{pl}}$ and $m = 10^{16} \text{ GeV} \sim 10^{-3}m_{\text{pl}}$. Both fields are normalized by m_{pl} . Although fluctuations grow at the initial stage, the maximum value $\sigma_{\text{max}} \approx 10^{-5}m_{\text{pl}}$ achieved in this case is much smaller than the value f . The ϕ_0 field evolves toward the potential minimum $\phi_0 = \pi f$, after which the universe enters the reheating stage. In this case, the enhancement of fluctuations hardly affects the evolution of the ϕ_0 field and the scale factor.
- FIG. 3: The evolution of ϕ_0 and σ fields for the initial value of $\phi(0) = 5.0 \times 10^{-7}m_{\text{pl}}$ in the case of $f = 10^{19} \text{ GeV} \sim m_{\text{pl}}$ and $m = 10^{16} \text{ GeV} \sim 10^{-3}m_{\text{pl}}$. The fluctuation σ reaches the maximum value $\sigma_{\text{max}} = 2.3m_{\text{pl}}$ when $\phi_0 = \pi f/2$. After that, σ decreases because spinodal instability is absent. The inflaton field finally rolls down toward the potential minimum with $\phi_0 = \pi f$ and $\sigma \approx 0$.
- FIG. 4: The evolution of ϕ_0 and σ fields for the initial value of $\phi(0) = 3.0 \times 10^{-7}m_{\text{pl}}$ in the case of $f = 10^{19} \text{ GeV} \sim m_{\text{pl}}$ and $m = 10^{16} \text{ GeV} \sim 10^{-3}m_{\text{pl}}$. The fluctuation σ reaches the maximum value $\sigma_{\text{max}} = 3.8m_{\text{pl}}$ when $\phi_0 = \pi f/2$. The secondary inflation due to fluctuations occurs for $1 \times 10^6 \lesssim mt \lesssim 7 \times 10^6$. In this region, since the effective two-field potential is very flat, the inflaton field evolves very slowly. Finally, inflaton is trapped in the potential minimum $\phi_0 = \pi f$ and $\sigma \approx 0$ at $mt = 7.6 \times 10^6$.
- FIG. 5: The evolution of the Hubble parameter H for the initial value of $\phi(0) = 3.0 \times 10^{-7}m_{\text{pl}}$ in the case of $f = 10^{19} \text{ GeV} \sim m_{\text{pl}}$ and $m = 10^{16} \text{ GeV} \sim 10^{-3}m_{\text{pl}}$. The first inflation occurs around $\phi_0 \approx 0$ for $0 \lesssim mt \lesssim 1.3 \times 10^5$, which is followed by the secondary inflation caused by fluctuations for $1 \times 10^6 \lesssim mt \lesssim 7 \times 10^6$. Since the duration of this secondary inflation is long, the achieved number of e -folding is very large as $N = 22178$.

Fig. 1

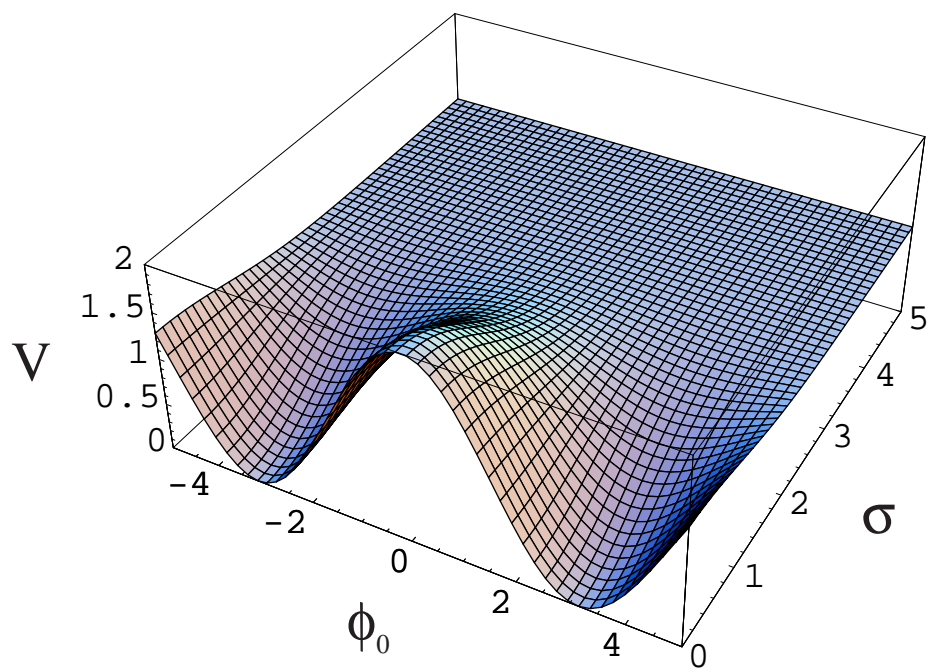


Fig. 2

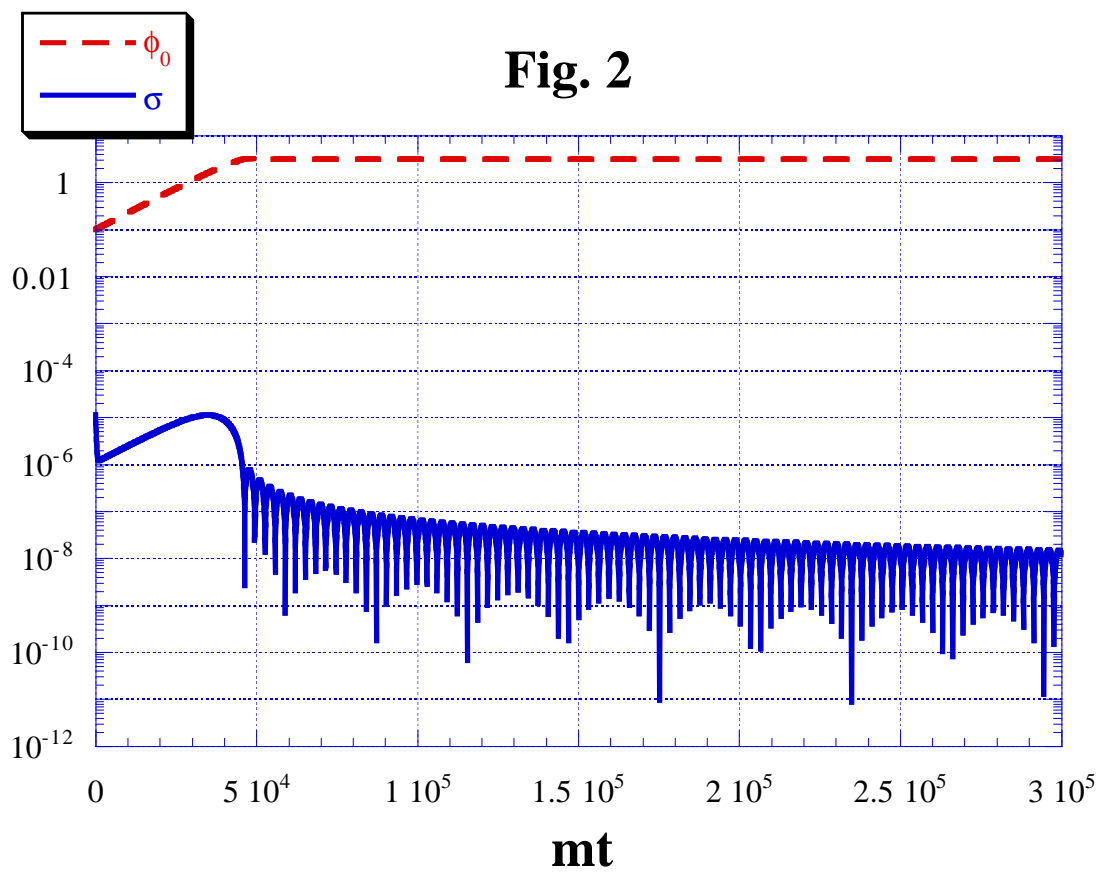


Fig. 3

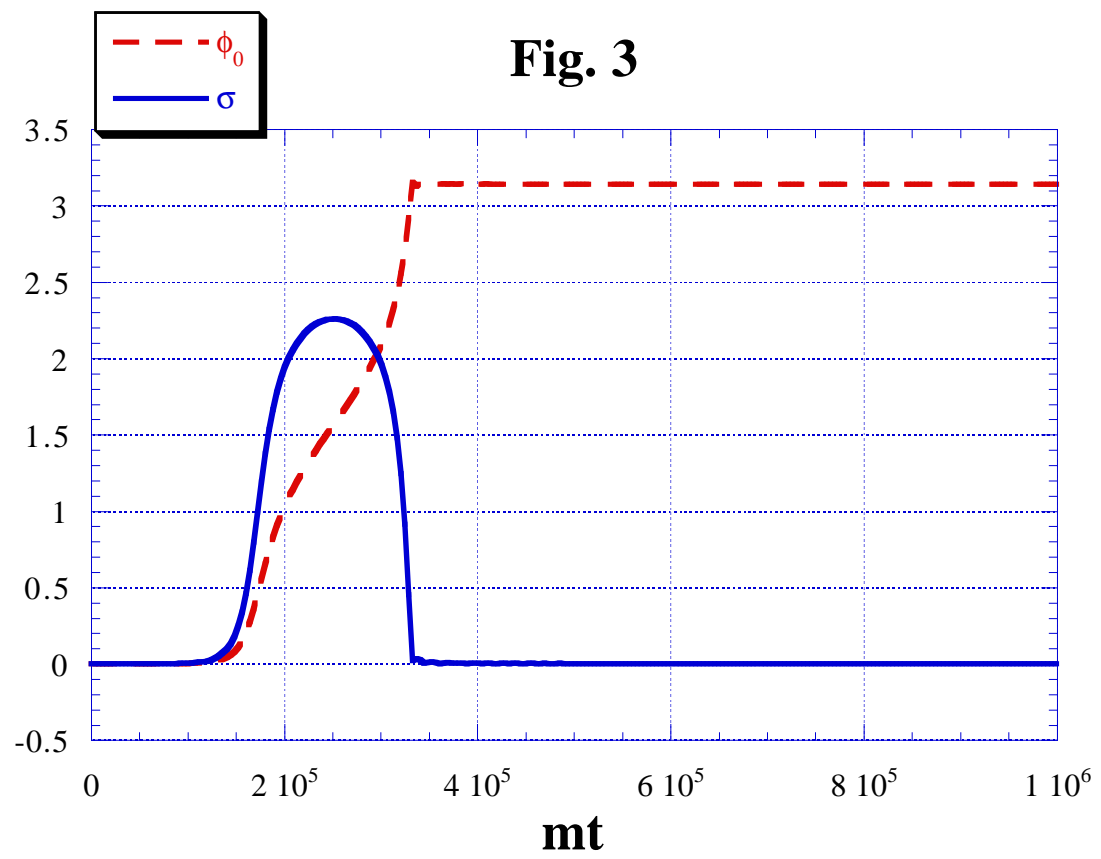


Fig. 4

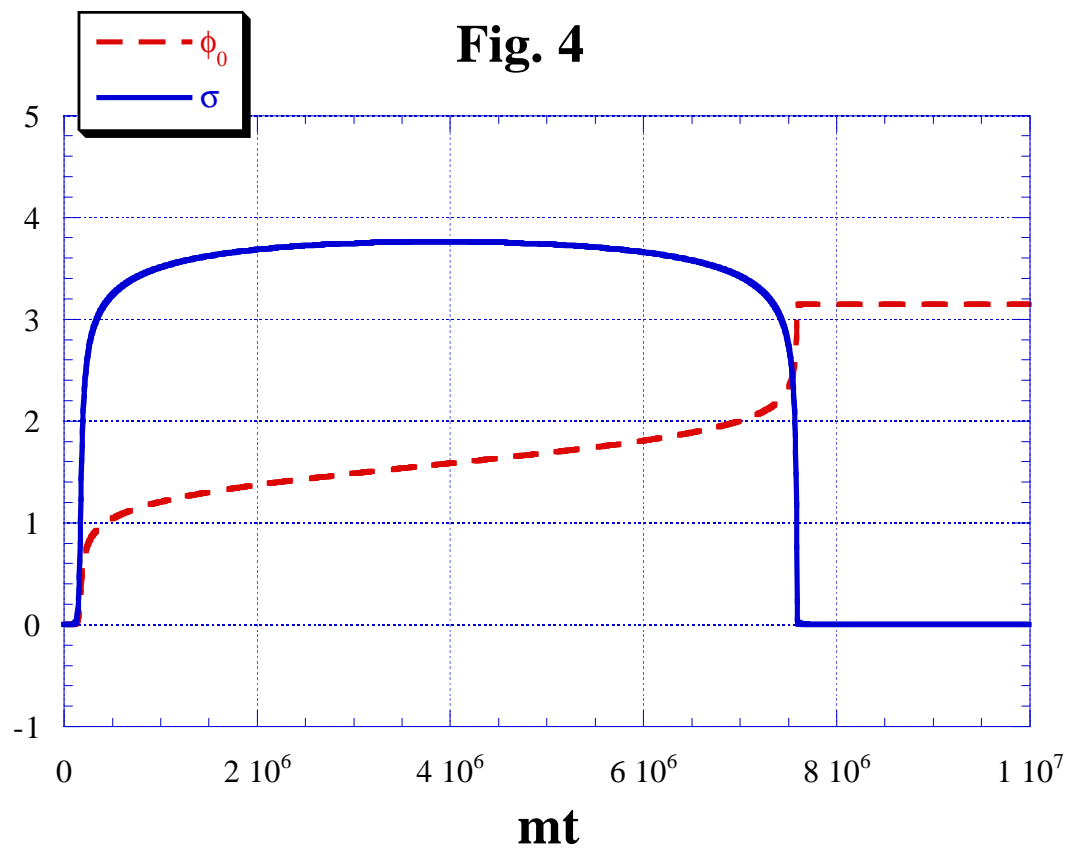


Fig. 5

